# Tutorial 9

## Exercises 16.3

19. $M = 3x^2,\ N = z^2/y,\ P = 2z\ln y$. $\frac{\partial M}{\partial y} = 0 = \frac{\partial N}{\partial x},\ \frac{\partial M}{\partial z} = 0 = \frac{\partial P}{\partial x},\ \frac{\partial N}{\partial z} = \frac{2z}{y} = \frac{\partial P}{\partial y}$

$\Rightarrow M\,dx + N\,dy + P\,dz$ is exact $\Rightarrow$ the potential function exists.

$\frac{\partial f}{\partial x} = 3x^2 \Rightarrow f(x,y,z) = x^3 + g(y,z) \Rightarrow \frac{\partial f}{\partial y} = \frac{\partial g}{\partial y} = z^2/y \Rightarrow g(y,z) = z^2\ln y + h(z)$

$\Rightarrow \frac{\partial f}{\partial z} = \frac{\partial g}{\partial z} = 2z\ln y + h'(z) = 2z\ln y \Rightarrow h'(z) = 0 \Rightarrow f(x,y,z) = x^3 + z^2\ln y + C$

$\Rightarrow \int_{(1,1,1)}^{(1,2,3)} 3x^2\,dx + z^2/y\,dy + 2z\ln y\,dz = f(1,2,3) - f(1,1,1) = 9\ln 2$

## Exercises 16.4

22. $M = 3y,\ N = 2x \Rightarrow \frac{\partial M}{\partial y} = 3,\ \frac{\partial N}{\partial x} = 2 \Rightarrow$ by Green's Theorem,

$\oint_C 3y\,dx + 2x\,dy = \iint_R (2-3)\,dx\,dy = \int_0^{\pi}\int_0^{\sin x} (-1)\,dy\,dx = -\int_0^{\pi} \sin x\,dx = \cos x\Big|_0^{\pi} = -2$

26. On the ellipse $r(t)$, $x = a\cos t,\ y = b\sin t \Rightarrow dx = -a\sin t\,dt,\ dy = b\cos t\,dt$

$\Rightarrow \text{Area } R = \frac{1}{2}\oint_C x\,dy - y\,dx = \frac{1}{2}\int_0^{2\pi} a\cos t \cdot b\cos t\,dt - b\sin t\cdot(-a\sin t)\,dt$

$= \frac{1}{2}\int_0^{2\pi} ab\,dt = ab\pi$